Gravitational Waves in Euclidean Space

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Abstract. Weak gravitational waves are described in Euclidean Space. Linear and non linear wave equations are derived. The non linear equation is solved and the result may be applied to the interpretation of cosmological data.

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INTRODUCTION

The Theory of Relativity is formally four dimensional and it might seem reasonable that waves related to a four dimensional environment should propagate in four dimensions. The usual analysis is mainly focused on treating gravitational waves like electromagnetic waves for what is related to the domain of propagation. The possibility that gravitational waves might travel in four dimensions, i.e. that they might travel also through proper time, is here explored theoretically, including some expected consequences.

The first step for a deeper understanding of the problem is to change from Lorentzian space-time coordinates to Euclidean coordinates (Montanus, 2001; Gersten, 2003). With the coordinate transformation, the universe is described with four space-like coordinates and an infinite number of "local" time variables. In Euclidean relativity time t goes back to its old status of being a running variable, not an explorable dimension, just like it once was in pre-relativistic physics, and every "object" has its own time variable, which sequentializes its activity and makes distinction among past, present and future for that specific object. The equivalent Euclidean form of the Theory of Relativity describes the behavior of light and near light-speed phenomena with the same accuracy as the usual Theory of Relativity.

Euclidean Relativity can be easily extended to a form of "brane" world multiverse theory that does not require more than the four dimensions of the so called "Euclidean Space" (Fontana, Murad and Baker, 2007), or of other extended space-time concepts (Froning, 2004; Meholic, 2004). The Euclidean Space hosts a parallel space-times structure on **each** of its four dimensions (Fontana, 2005), this structure is compatible with the existence of "non visible" matter and predicts it in the amount measured through gravitational effects by just assuming the other three space-time structures are similar to our own (Fontana, 2005).

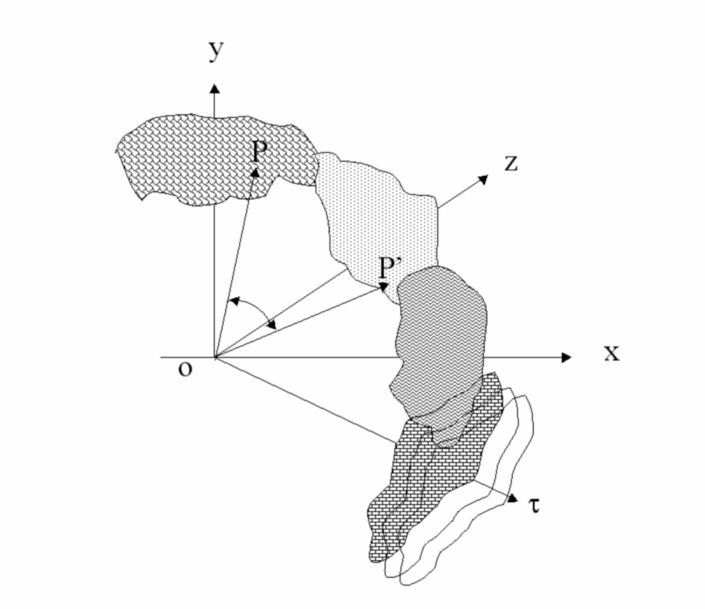

FIGURE 1. Parallel space-times (parallel worlds) along τ, ortogonal space-times (dark universes) along x, y, z.

Let's say that the structure of flat space-time is the following Minkowski metric:

$$(d\tau)^2 = c^2(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2, \tag{1}$$

which introduces the speed of light c and its properties in the tri-dimensional space and time. Rearranging we have:

$$c^2(dt)^2 = (d\tau)^2 + (dx)^2 + (dy)^2 + (dz)^2. \tag{2}$$

In Equation (2) τ, x, y and z are the coordinates of the Euclidean Space and t is the variable used to evaluate velocity and acceleration. Time t is an integral local function of changes of the four space coordinates. Motion in the Euclidean Space is the origin of time t, and the origin of our view of reality. Equation (2) in not a metric in the usual relativistic interpretation, it only indicates a property of observable particles as deduced from experiments in a space-time. Particles that obey Lorentz invariance, also obey:

$$\left(\frac{d\tau}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = c^2, \tag{3}$$

obtained from Equation (2) by dividing by $(dt)^2$; those particles belong to a space-time. The speed c is an invariant. Returning to Equation (1), it can be rewritten as:

$$(d\tau)^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu, \tag{4}$$

$\eta_{\mu\nu}$ is the Minkowski metric in matrix form (Weinberg, 1972):

$$\eta_{\mu\nu} = \begin{vmatrix} c^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}. \tag{5}$$

A generic low amplitude gravitational wave, fully consistent with Einstein field equations is described by a perturbation of the Minkowski metric, i.e. $h_{\mu\nu}$<<1:

$$(d\tau)^2 = g_{\mu\nu}dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \ . \tag{6}$$

For a plane gravitational wave traveling along the *z* axis at the speed of light in flat space-time it is found that only the components of the metric along *x* and *y* may change because of the passage of the gravitational wave. With *a* and *b* solutions of a generic wave equation, the symmetric tensor for the polarizations + and × is:

$$h_{\mu\nu} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}. \tag{7}$$

GRAVITATIONAL WAVES IN THE EUCLIDEAN SPACE

The conversion of the weak gravitational wave metric to Euclidean form is performed on the above mentioned example in order to gain some information on the behavior of gravitational waves in the Euclidean Space. A generalization by symmetrization will follow and additional examples are reserved for future papers.

Writing Equation (6) and Equation (7), with $x_0..x_3$ to be *t, x, y, z* respectively and by keeping only the polarization aligned with *x* and *y* we have:

$$(d\tau)^2 = c^2(dt)^2 - (1-a)(dx)^2 - (1+a)(dy)^2 - (dz)^2. \tag{8}$$

In Euclidean form it is:

$$c^2(dt)^2 = (d\tau)^2 + (1-a)(dx)^2 + (1+a)(dy)^2 + (dz)^2. \tag{9}$$

The simplest method to describe the interaction between a weak gravitational wave and particles traveling in the Euclidean Space is by writing the speed invariant of Equation (3) with the gravitational wave.

The Euclidean Space four-speed invariant including a plane gravitational wave with + polarization traveling along z is therefore:

$$\left(\frac{d\tau}{dt}\right)^2 + (1-a)\left(\frac{dx}{dt}\right)^2 + (1+a)\left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = c^2. \tag{10}$$

We easily recognize that photons traveling at the speed of light along z with $d\tau/dt = 0$ have their trajectory deflected along *x* and *y*. For masses in our space-time we have $d\tau/dt \approx$ c and $dz/dt \approx 0$ and their four-speed similarly modulated. Both must have non-zero initial four-speed along x and y in the Euclidean Space in order to have it modulated by the gravitational wave, thus posing a question of detectability in some specific states of motion in the Euclidean Space. For the wave interaction described by Equation (10), a mechanical constraint regarding *dx/dt* or *dy/dt* may induce changes in the remaining two components of the four-speed. Future work will analyze the details of detectors of gravitational waves in the Euclidean Space framework.

According to Fontana (2005) four orthogonal space-times coexist in every infinitesimal volume of Euclidean Space and the role of photons and matter can be interchanged in Equation (10) if a different orthogonal space-time is considered. To be consistent with the speed invariance and the symmetry principles that led to the Euclidean Space concept (Modanese and Fontana, 2004; Fontana, 2005; Fontana, Murad and Baker, 2007), gravitational waves are allowed to travel with components along the four coordinates of the Euclidean Space. Each component involves two coordinates and affects matter that belongs to space-times traveling along both of the remaining two, including their respective photons.

Both in General Relativity and Euclidean Relativity, matter and waves "fill" the respective four dimensional spaces and are therein described mathematically in their whole cosmological history. Euclidean Relativity advantageously decouples the "content" of "space", which is now a four dimensional pure space/Euclidean Space or "memory space", from the "editing processes" represented by the parallel brane/space-times passages or sweeps, driven by quantum rules in conjunction with their speed invariant (Fontana, 2006).

The Hyperspherical symmetry (Almeida, 2004) has been invoked to model a **uniformly expanding** Universe, while the observed accelerating expansion will be discussed later in this paper.

PROPAGATION OF GRAVITATIONAL WAVES

Gravitation can be easily introduced in the Euclidean theory by resorting to the analogy to optical propagation in a three-space and introducing a gravitational refractive index of the Euclidean Space named n (Almeida, 2001):

$$c^2(dt)^2 = n^2\left[(d\tau)^2 + (dx)^2 + (dy)^2 + (dz)^2\right], \quad n \neq 0, \tag{11}$$

after switching to spherical coordinates with $d\varphi=0$ and $d\theta=0$, writing $ds^2=n(d\tau^2)$ and substituting:

$$\frac{1}{n} = \left(1 - \frac{2Gm}{r}\right), \tag{12}$$

we find the Schwarzschild's metric of General Relativity (Almeida, 2001), with G the gravitational constant, m is mass and r distance. Therefore there exist a formal link between General Relativity and the Theory of Gravity for the Euclidean Space that is called Four Dimensional Optics (4DO).

For the wave described by Equation (11), with $a<<1$ we express Equation (11) as:

$$c^2(dt)^2 = n_\tau^2(d\tau)^2 + n_x^2(dx)^2 + n_y^2(dy)^2 + n_z^2(dz)^2\,, \tag{13}$$

Comparing Equation (10) with Equation (13), it is obvious that the weak gravitational wave is a wave of refractive index of the Euclidean Space, which propagates at the speed of light in four dimensions. More generally, assuming isotropic propagation, a generic wave equation for weak waves traveling at speed c can be written:

$$\frac{1}{c^2}\frac{\partial^2 n}{\partial t^2} = \nabla^2 n\,, \tag{14}$$

Equation (14) is characterized by time t defined as an evolution variable as in the equations of electrodynamics and by having four space dimensions.

The linear wave Equation (14) can be integrated with results that will have integration constants. In fact in the flat background, the refractive index is unitary; this value must be added to the solutions of Equation (14).

If the background value can be ignored in the derivatives, it must be included in the expression of speed. The nonlinear wave equation that includes the effect of the refractive index on speed is therefore:

$$\frac{(n+1)^2}{c^2}\frac{\partial^2 n}{\partial t^2} = \nabla^2 n\,, \tag{15}$$

whose nonlinear behavior expresses the non-linearity of gravitational waves (Christodoulou, 1991). The background value of *n* must be added to the solutions of Equation (15).

Equation (14) has the usual wave solutions for any value of *n*, instead Equation (15) has the usual wave solutions if $n^2 <<1$.

The solutions for weak waves in a unitary background may come from separations of variables in spherical (hyperspherical) coordinates. Writing the solutions of Equation (14) in the form *n(r,t)= T(t) R(r)* , the individual equations are:

$$r\frac{d^2 R}{dr^2} + (i-1)\frac{dR}{dr} + k^2 rR = 0\,; \qquad \frac{d^2 T}{dt^2} + \omega^2 T = 0\,, \tag{16}$$

where *i* is the space dimensionality of the problem, *k* has units of 1/distance and ω has units of 1/time. With *i=4*, they are waves in four dimensions. Please note that with odd dimensionality (*i*= 1, 3, 5, …) the traveling wave solutions have speed *c*; instead for even dimensionality the traveling wave solutions have a spread of speeds from zero to a maximum (Brown, 2007). This last result applies to gravitational waves in the Euclidean Space.

Qualitatively, according to Equation (15), if a beam of gravitational waves is more intense at the center respect to the periphery, then the central part of the beam travels at lower four-speed, leading to self-focusing, in agreement with the classical treatment of the problem (Corkill and Stewart, 1983; Ferrari,1988).

The unitary background refractive index of Equation (15) needs some explanation as it could be attributed to a background gravitational wave that cosmologically supports our space-time. In fact removing the background the non-linear wave equation is:

$$\frac{n^2}{c^2}\frac{\partial^2 n}{\partial t^2} = \nabla^2 n\,, \tag{17}$$

Starting from the wave equation in *i* space dimensions (*i*=4 for the Euclidean Space) and adopting again *n(r,t)= T(t) R(r)*:

$$T\frac{d^2 R}{dr^2} + (\frac{i-1}{r})T\frac{dR}{dr} = R\frac{R^2 T^2}{c^2}\frac{d^2 T}{dt^2}\,, \tag{18}$$

we separate the variables:

$$\frac{1}{R^3}\frac{d^2 R}{dr^2} + \frac{1}{R^3}(\frac{i-1}{r})\frac{dR}{dr} = \kappa = \frac{T}{c^2}\frac{d^2 T}{dt^2}\,, \tag{19}$$

rewriting, the left hand equation is:

$$r\frac{d^2R}{dr^2}+(i-1)\frac{dR}{dr}-\kappa\, r\, R^3=0\,, \tag{20}$$

while the right hand equation gives:

$$\frac{d^2T}{dt^2}=\frac{\kappa\, c^2}{T}\,, \tag{21}$$

that is a case of the Emden-Fowler (EF) nonlinear ordinary differential equation with $A=\kappa\, c^2$ (Sachdev, 1991). Equation (21) has no known explicit solution in the form *T(t)=f(t)*, instead it is possible to write:

$$t=\int\left(2A\ln T+C_2\right)^{-1/2}dT+C_1\,. \tag{22}$$

With $\kappa\neq0$, Equation (21) can be integrated numerically with the approximate result: $T(t)\approx C_1+C_2t^{\alpha}$, with C_1 and C_2 real constants and α slightly larger than 1 ($\alpha\approx1.03$). The solutions with both $C_1\neq0$ and $C_2=0$ are not acceptable, because *T(t)=constant* does not satisfy Equation (21). With numerical integration, the slope and its sign depend on the initial conditions of the integrators, the equation is obviously singular for T=0. Other cases of the EF equation, which are numerically treatable like this one, do have explicit analytical solutions; those have been used for testing the numerical integrator and to find the class of functions for fitting the parameters of the approximate solution. With $\kappa=0$ the solution is $T(t)= C_1+C_2t$. As κ has units of *1/distance*^{2}, the physical significance of such a situation has to be investigated, but we observe that with $\kappa=0$, the solution *T(t)=constant* is consistent with a stationary Euclidean Space. With negative κ the spatial term of Equation (16) is a damped oscillation, the same we have for Equation (20).

The evolution of space-times within the cosmological non-linear gravitational wave can be interpreted by saying that the large scale sweeping of space-times through the Euclidean Space may become slower and slower as long as time *t* passes, on the other hand, in spite of the increasing temporal component of *n*, the oscillating and decreasing spatial component of *n* combined with the motion of a specific space-time may also result in a nearly constant *n* for the local background, during some evolution stage of the specific space-time. In addition the background value of the refractive index *n(r,t)= T(t) R(r)* depends on *r* through Equation (20), therefore we may have regions of the Euclidean Space in which the propagation speed of all particles is different from *c*, and the existence of Euclidean Space "highways" is not excluded, especially if the complete solution to the spatial equation including angles is considered. The discovery of those regions may be of great importance for interstellar travel. In the Euclidean Space, *n* can oscillate from positive to negative values. The effect of the sign change on space-times has to be investigated. As pointed out by Murad (2007), with $\kappa=0$ the solution of the spatial equation is simply $R(r)=(1-i)\ln(r)+C$, therefore $n(r,t)= (C_1+C_2t)[(1-i)\ln(r)+C_3]$; again this particular solution indicates that the speed of space-times in the Euclidean Space changes because of evolution in time and expansion in space.

The existence of a cosmological primordial gravitational wave may affect, particularly with its **space component** *R(r)*, the expansion speed of space-times as shown by the following picture, thus improving Almeida's geometrical model of the expansion of the universe:

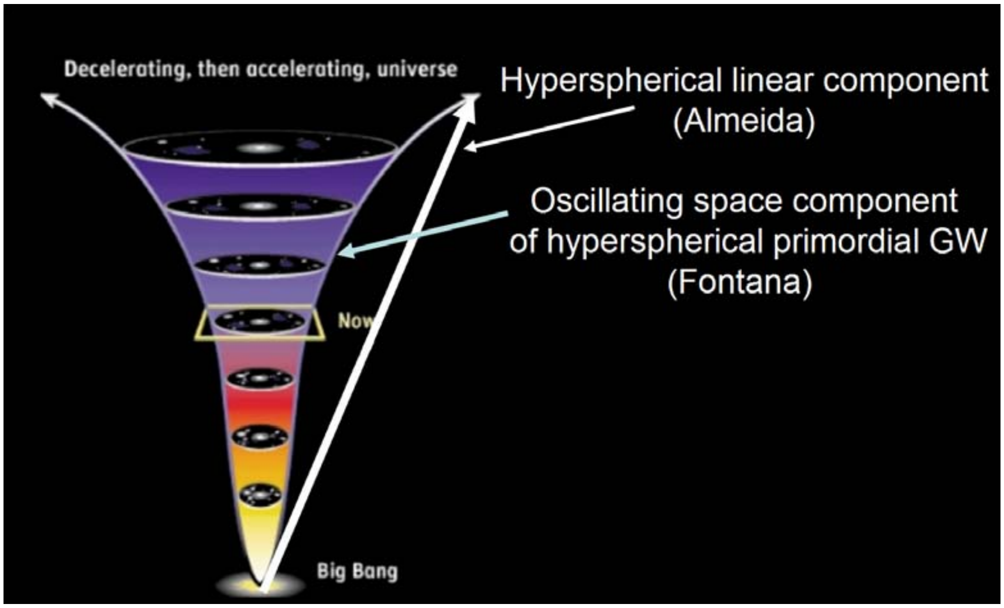

FIGURE 2. Expansion of our space-time with variable speed.

CONCLUSION

The Euclidean Space approach to gravitational waves has allowed an exact solution to the hyperspherical non linear gravitational wave. It may also offer some interesting inputs to cosmology as soon as it is recognized that the Euclidean Space itself might be a special gravitational wave. Non-baryonic dark matter is here associated to matter belonging to orthogonal universes and this model can predict it in the observed amount. In an attempt to complete the geometric approach to the modelization of the expansion of the universe, the present analysis suggests that a primordial/cosmological non linear gravitational wave may modulate the expansion speed, therefore causing the observed acceleration and deceleration of cosmic expansion.

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