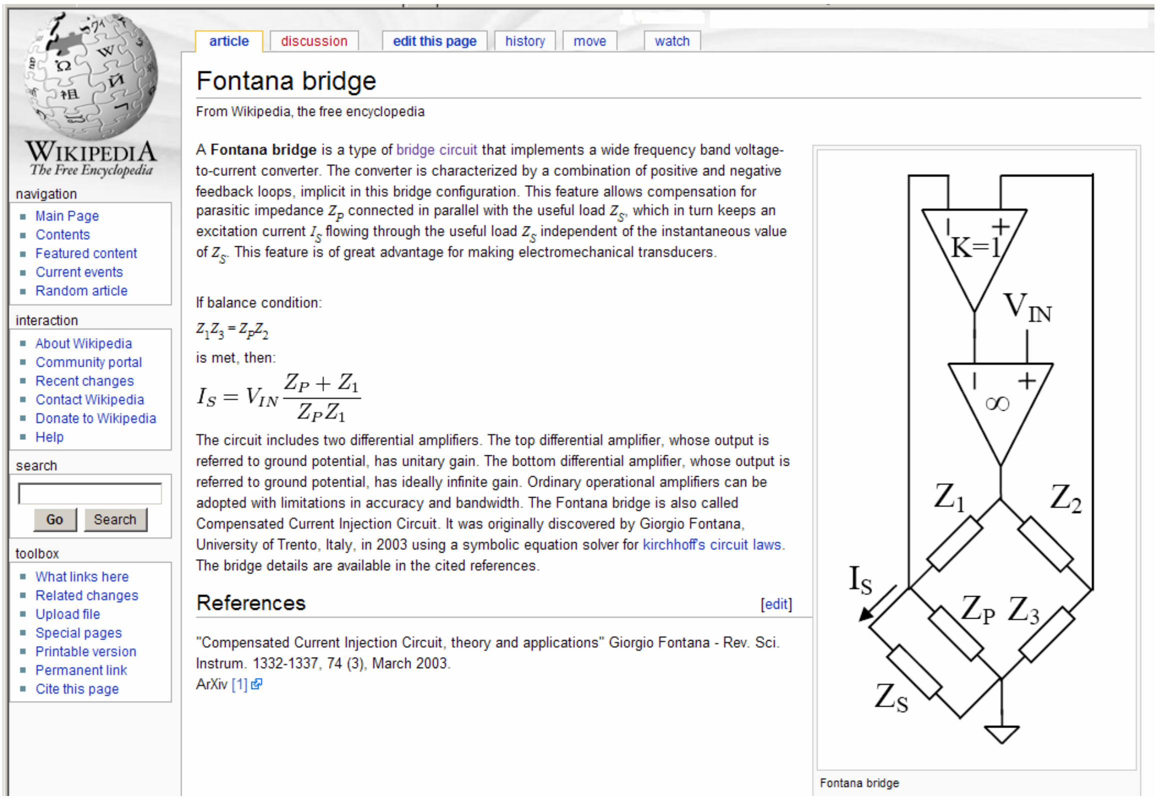
Step by step solution of the Fontana bridge.

Kindly provided by G. Fontana himself
on July 2008.



Fontana bridge

From Wikipedia, the free encyclopedia

A **Fontana bridge** is a type of bridge circuit that implements a wide frequency band voltage-to-current converter. The converter is characterized by a combination of positive and negative feedback loops, implicit in this bridge configuration. This feature allows compensation for parasitic impedance Z_P connected in parallel with the useful load Z_S, which in turn keeps an excitation current I_S flowing through the useful load Z_S independent of the instantaneous value of Z_S. This feature is of great advantage for making electromechanical transducers.

If balance condition:

$Z_1 Z_3 = Z_P Z_2$

is met, then:

$$I_S = V_{IN}\frac{Z_P + Z_1}{Z_P Z_1}$$

The circuit includes two differential amplifiers. The top differential amplifier, whose output is referred to ground potential, has unitary gain. The bottom differential amplifier, whose output is referred to ground potential, has ideally infinite gain. Ordinary operational amplifiers can be adopted with limitations in accuracy and bandwidth. The Fontana bridge is also called Compensated Current Injection Circuit. It was originally discovered by Giorgio Fontana, University of Trento, Italy, in 2003 using a symbolic equation solver for kirchhoff's circuit laws. The bridge details are available in the cited references.

References [edit]

"Compensated Current Injection Circuit, theory and applications" Giorgio Fontana - Rev. Sci. Instrum. 1332-1337, 74 (3), March 2003.
ArXiv [1]

Let's name *V-* the inverting input of the top differential amplifier and *V+* the non inverting input of the top differential amplifier, let's name *Vout* the output of the bottom differential amplifier. The steps are:

1) Compute *V-* as a function of *Vout* using the voltage divider formula:

$$V_- = V_{out}\frac{\dfrac{Z_P Z_S}{Z_P + Z_S}}{Z_1 + \dfrac{Z_P Z_S}{Z_P + Z_S}} = V_{out}\frac{Z_P Z_S}{Z_P Z_S + Z_P Z_1 + Z_S Z_1}$$

2) Compute *V+* as a function of *Vout*:

$$V_+ = V_{out}\frac{Z_3}{Z_2 + Z_3}$$

3) Compute *Is* as a function of *V-* and, using 1), as a function of *Vout* :

$$I_S = \frac{V_-}{Z_S} = V_{out} \frac{Z_P}{Z_P Z_S + Z_P Z_1 + Z_S Z_1}$$

4) Taking into account the infinite gain of the bottom differential amplifier, we have for a stable circuit:

$$V_+ - V_- = V_{IN}$$

5) Find the relation between V_{IN} and *Vout* by combining 1), 2) and 4).

$$V_+ - V_- = V_{IN} = V_{out}\left(\frac{Z_3}{Z_2 + Z_3} - \frac{Z_P Z_S}{Z_P Z_S + Z_P Z_1 + Z_S Z_1}\right)$$

$$V_{out} = V_{IN} \frac{1}{\frac{Z_3}{Z_2 + Z_3} - \frac{Z_P Z_S}{Z_P Z_S + Z_P Z_1 + Z_S Z_1}}$$

6) From 3) and 5) we have:

$$I_S = V_{IN} \frac{1}{\frac{Z_3}{Z_2 + Z_3} - \frac{Z_P Z_S}{Z_P Z_S + Z_P Z_1 + Z_S Z_1}} \cdot \frac{Z_P}{Z_P Z_S + Z_P Z_1 + Z_S Z_1}$$

$$I_S = V_{IN} \frac{(Z_2 + Z_3)(Z_P Z_S + Z_P Z_1 + Z_S Z_1)}{Z_3(Z_P Z_S + Z_P Z_1 + Z_S Z_1) - Z_P Z_S (Z_2 + Z_3)} \cdot \frac{Z_P}{Z_P Z_S + Z_P Z_1 + Z_S Z_1}$$

$$I_S = V_{IN} \frac{Z_P (Z_2 + Z_3)}{Z_3(Z_P Z_S + Z_P Z_1 + Z_S Z_1) - Z_P Z_S (Z_2 + Z_3)}$$

7) Make the products:

$$I_S = V_{IN} \frac{Z_P Z_2 + Z_P Z_3}{Z_3 Z_P Z_S + Z_3 Z_P Z_1 + Z_3 Z_S Z_1 - Z_P Z_S Z_2 - Z_P Z_S Z_3}$$

8) Simplify:

$$I_S = V_{IN} \frac{Z_P Z_2 + Z_P Z_3}{Z_3 Z_P Z_1 + Z_3 Z_S Z_1 - Z_P Z_S Z_2}$$

9) Collect terms and find the balance equation:

$$I_S = V_{IN} \frac{Z_P Z_2 + Z_P Z_3}{Z_3 Z_P Z_1 + Z_S (Z_3 Z_1 - Z_P Z_2)}$$

By inspection, the balance equation is: $Z_3 Z_1 = Z_P Z_2$

10) Apply the balance equation:

$$I_S = V_{IN} \frac{Z_P Z_2 + Z_P Z_3}{Z_3 Z_P Z_1} = V_{IN} \frac{Z_2 + Z_3}{Z_3 Z_1}, \ with \quad Z_P \neq 0$$

11) Observe that:

$$Z_3 Z_1 = Z_P Z_2 \ implies \ \frac{Z_3}{Z_P} = \frac{Z_2}{Z_1}, \ with \quad Z_P \neq 0 \quad and \quad Z_1 \neq 0$$

12) Using them we rewrite:

$$I_S = V_{IN} \frac{Z_2 + Z_3}{Z_3 Z_1} = V_{IN} \frac{Z_1 \frac{Z_3}{Z_P} + Z_P \frac{Z_2}{Z_1}}{Z_1 Z_P \frac{Z_2}{Z_1}} = V_{IN} \frac{Z_1 \frac{Z_2}{Z_1} + Z_P \frac{Z_2}{Z_1}}{Z_1 Z_P \frac{Z_2}{Z_1}} = V_{IN} \frac{Z_1 + Z_P}{Z_1 Z_P}$$

Great, now you are ready for your application.