Use of a Notch Filter in a Tuned Mode for LISA.

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Abstract. During interferometric measurements the proof mass must be free from any controlling force within a given observation bandwidth. This requirement imposes the use of high slope filters in the feedback loop, rising problems of stability. Here a tuned mode, decoupled with a notch filter is presented. This mode guarantees stability at any possible DC open loop gain. Operationally, the tuned mode is similar to the narrow band radio receiver way of operation.
How the servo signal is transferred to the proof mass (forces applied to the proof mass or activation of spacecraft thrusters or both) is not considered here.

Basic concept of stability in a feedback loop.

We recall the Bode stability criterion:
For the feedback system like that of fig. 1, the open loop transfer function is G, the closed loop transfer function is W = G/(1+GH), the loop transfer function is -GH.

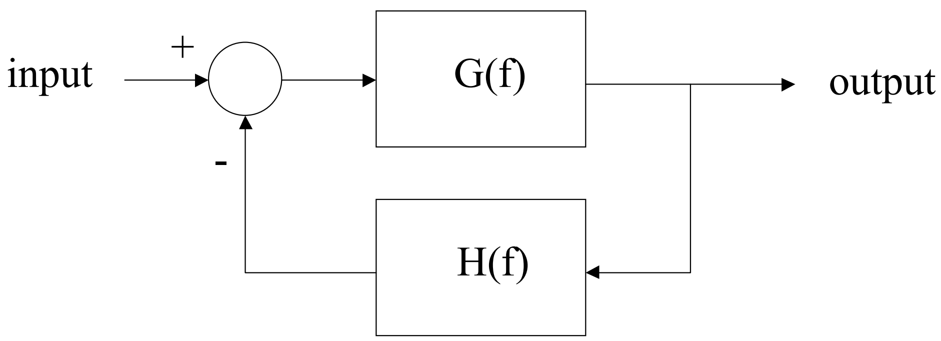


Figure 1: a typical feedback system.

Here G and H are complex functions of frequency and they are characterised by mudulus and phase. Not all possible G and H are well behaved, in fact if there exist a frequency for which the modulus of GH is one and the phase of GH is $\pi + 2N\pi$, N=-1,0,1,2,… then we have an oscillator. The condition for the modulus has to be extended to greater and equal than one, because nonlinearities are capable of reducing the gain for a sufficiently high amplitude of the controlled quantity.
Taking into account that for physical systems (minimum delay systems) modulus and phase are linked by an integral function, we can define and use a quite general example of a loop transfer function. A stability criterion for GH is therefore shown in fig. 2

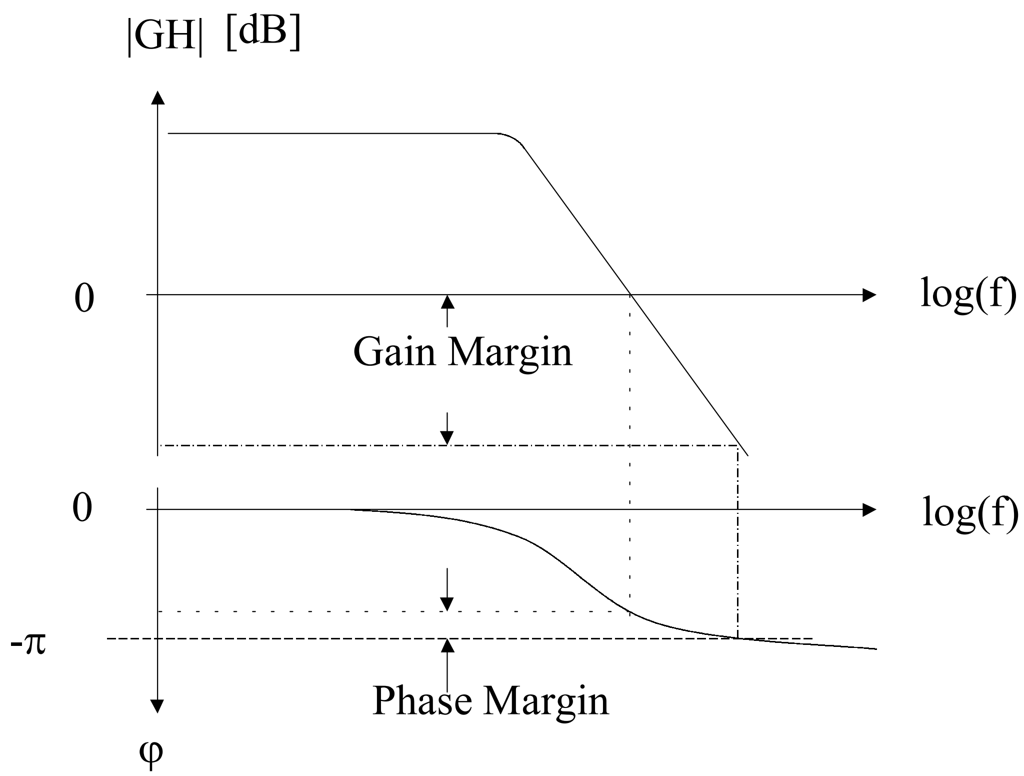


Figure 2. A stability criterion for the modulus and the phase of GH.
The quality of the stability can also be determined.

The design of a stable feedback system is something like to determine, if possible, the transfer function of a "controller" G' that is inserted after G so that GG'H is like the one shown in fig. 2. The transfer function becomes W=GG'/(1+GG'H).
If GG'H is much greater than one W≈1/H.
This technique is employed in the vast majority of the so called operational amplifiers. More complex and effective control techniques have been developed, but we do not need them in this example.

How to open the loop at a given frequency.

The feedback loop can be made ineffective at a given and tunable frequency with a notch filter with transfer function F. The phase lag of the double T notch filter is always between $-\pi/2$ and $\pi/2$, and if it operates at a frequency where the modulus of GG'H is flat, the phase lag will never be sufficient to induce instability. The transfer function becomes W=GG'F/(1+GG'FH). If GG'FH is much greater than one then W≈1/H otherwise we have W≈ GG'F, making the input electrically unobservable at the notch frequency. Obviously no feedback is applied at the notch frequency.

The double T Notch filter transfer function.

The schematic of the notch filter is presented in fig. 3.

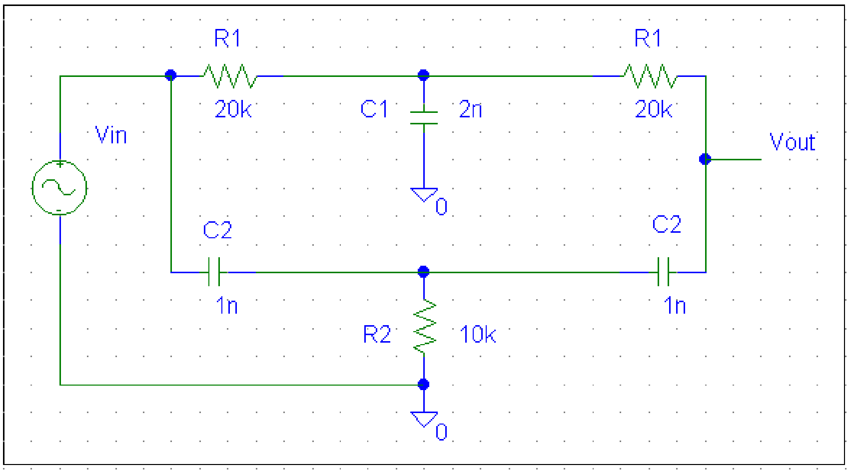


Figure 3. The notch filter.

If C1=2 C_2 and R1=2 R_2, the transfer function is:

$$F = \frac{1-(\omega R_1 C_2)^2}{1 + j\omega 4 R_1 C_2 - (\omega R_1 C_2)^2} \qquad 1)$$

The notch frequency is $\omega_0=1/(R_1C_2)$.
The transfer function of the filter is obviously causal and can be synthesised numerically.
The modulus and the phase of the transfer function have been computed numerically with the component values shown in fig 3. Using the electronic circuit simulator PSPICE.

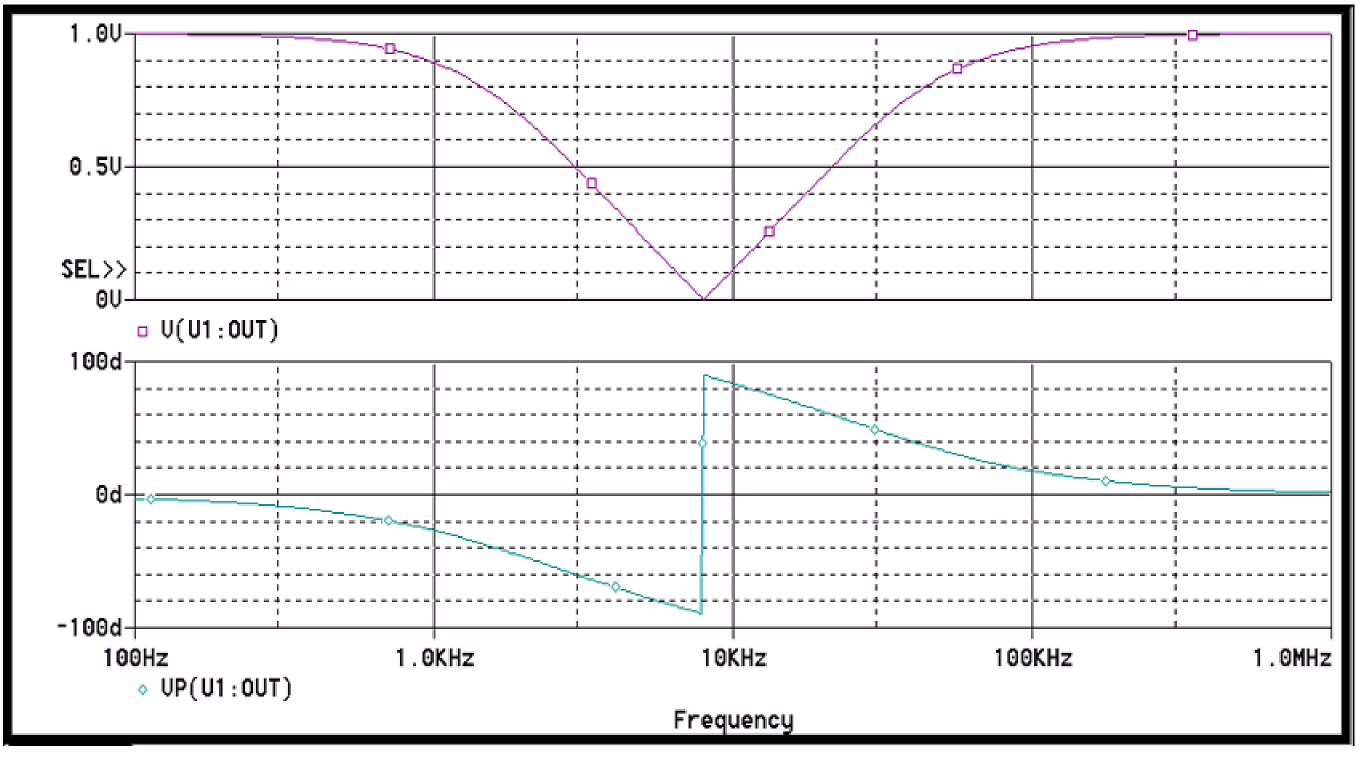


Figure 4. Modulus and phase response of the notch filter, phase in degrees.

It is the numerator of the fraction in Eq. 1 that determines the attenuation behaviour of the notch filter. For an attenuation of more than 100 dB (10^5 times less gain) the useful (decoupled) bandwidth is the notch filter frequency divided by about a factor of 10^5.

The following plot shows the transfer function of a first order low pass filter and the notch filter in series.

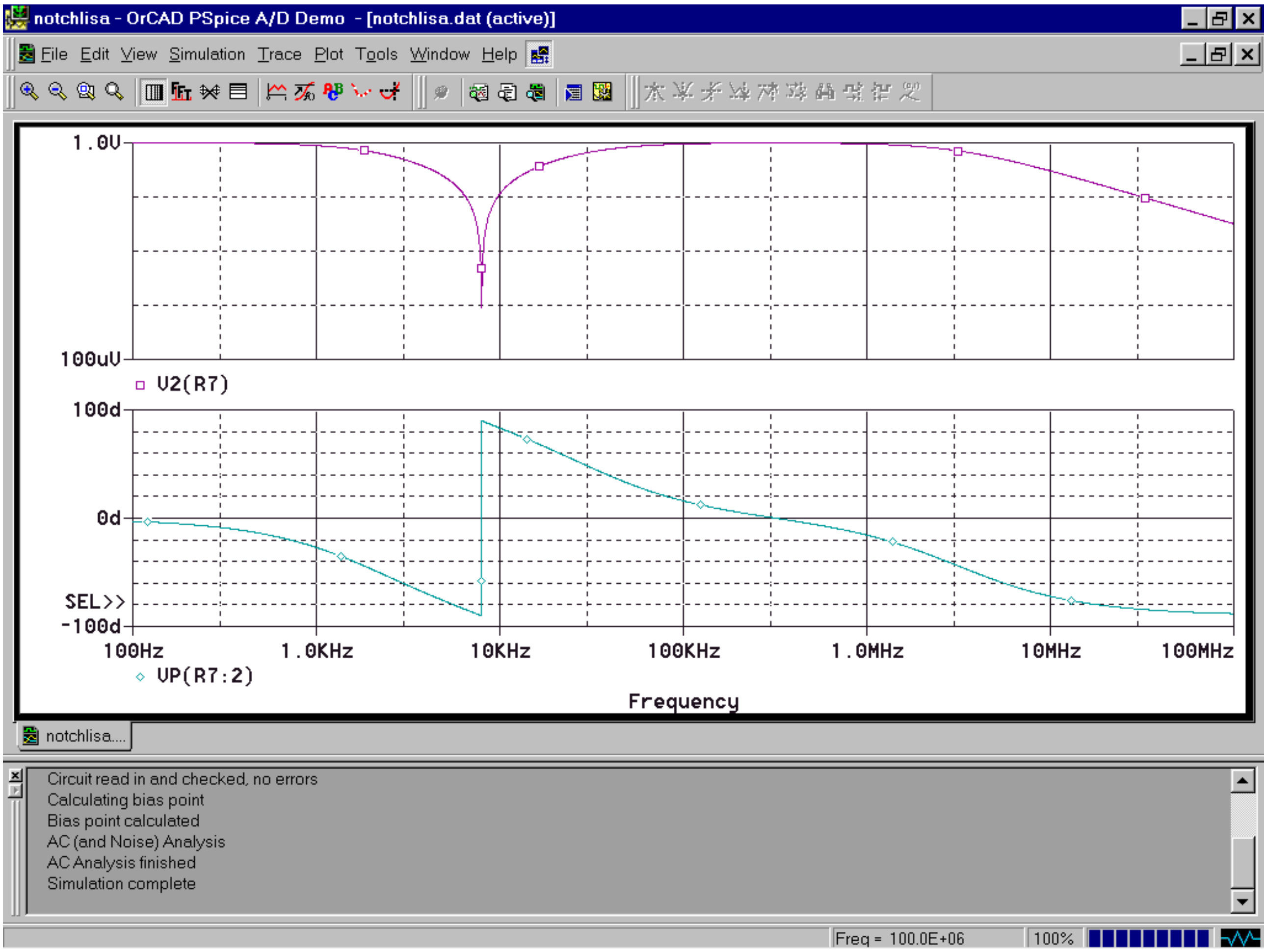


Figure 5. Modulus and phase response of a L.P + notch filter, phase in degrees.

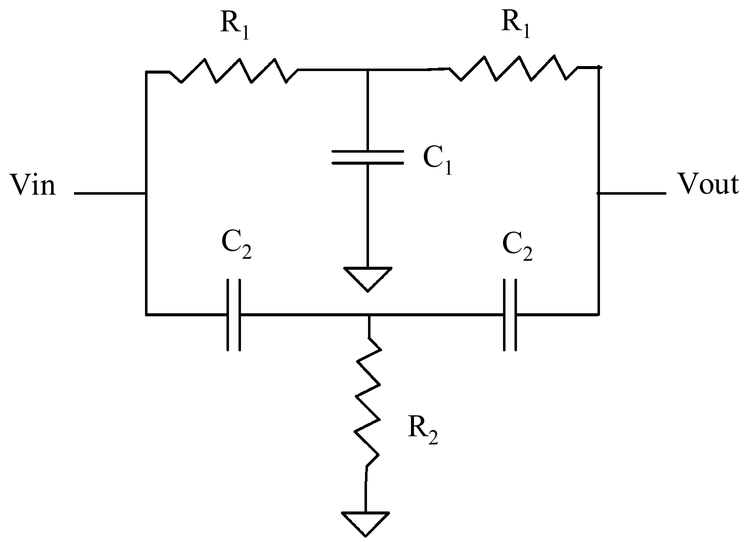
In the arbitrary frequency range chosen, we observe that the notch filter also improves the phase margin in the high frequency range. The phase is always well behaved.

Conclusion.

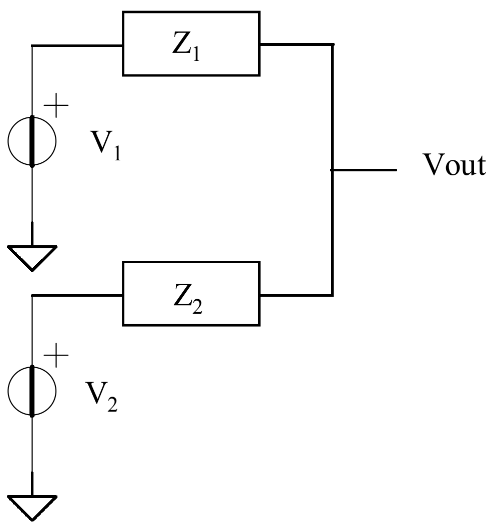
It has been shown that an RC Notch filter or its numerically synthesised version can be inserted in a feedback loop with the purpose of inactivating the loop at a given changeable frequency.

APPENDIX: calculation of the double T Notch filter transfer function.

The transfer function of the double T notch filter can be calculated using Thevenin and Norton equivalent circuits. This procedure is simpler than writing Kirkhhoff equations and solving the system of equations.
The double T notch filter is:



We can define two Thevenin equivalent circuits for the two “T”.



Open circuit equivalent voltage V_1 can be computed taking into account left R_1 and C_1; right R_1 has no voltage drop because we are evaluating an open circuit voltage.

$$V_1 = V_{IN} \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = V_{IN} \frac{1}{1 + j\omega R_1 C_1} \qquad 1)$$

Output impedance Z1 can be computed evaluating the impedance of left R_1,C_1 and right R_1 network, as seen from its output with the voltage generator V_{in} switched off, i.e. shorted.

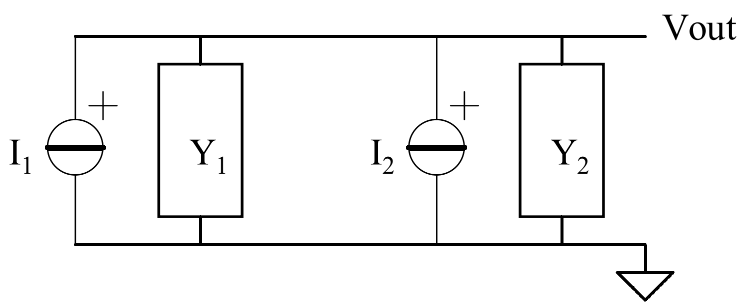
$$Z_1 = R_1 + \frac{R_1}{1 + j\omega R_1 C_1} \qquad 2)$$

The same procedure applies for deriving V_2 and Z_2.

$$V_2 = V_{IN} \frac{R_2}{R_2 + \frac{1}{j\omega C_2}} = V_{IN} \frac{j\omega R_2 C_2}{1 + j\omega R_2 C_2} \qquad 3)$$

$$Z_2 = \frac{1}{j\omega C_2} + \frac{R_2}{1 + j\omega R_2 C_2} \qquad 4)$$

It is now necessary to switch to a couple of Norton equivalent circuits in order to calculate the parallel of the two “T” networks.



The conversion rule between Z (impedance) and Y (admittance) is Y=1/Z, we also have I=V/Z

For network 1 we can write:

$$Z_1 = R_1 + \frac{R_1}{1 + j\omega R_1 C_1} = \frac{2R_1 + j\omega {R_1}^2 C_1}{1 + j\omega R_1 C_1} = \frac{R_1(2 + j\omega R_1 C_1)}{1 + j\omega R_1 C_1} \qquad 5)$$

$$Y_1 = \frac{1 + j\omega R_1 C_1}{R_1(2 + j\omega R_1 C_1)} \qquad 6)$$

$$I_1 = V_{IN}\frac{\frac{1}{1+j\omega R_1C_1}}{\frac{R_1(2+j\omega R_1C_1)}{1+j\omega R_1C_1}} = V_{IN}\frac{1}{R_1(2+j\omega R_1C_1)} \qquad 7)$$

The same procedure applies to network 2.

$$Z_2 = \frac{1+2j\omega R_2C_2}{j\omega C_2(1+j\omega R_2C_2)} \qquad 8)$$

$$Y_2 = \frac{j\omega C_2(1+j\omega R_2C_2)}{1+2j\omega R_2C_2} \qquad 9)$$

$$I_2 = V_{IN}\frac{-\omega^2 R_2C_2^{\,2}}{1+2j\omega R_2C_2} \qquad 10)$$

We can now add currents and admittances:

$$I = I_1 + I_2 = V_{IN}\left(\frac{1}{R_1(2+j\omega R_1C_1)} - \frac{\omega^2 R_2C_2^{\,2}}{1+2j\omega R_2C_2}\right) \qquad 11)$$

$$Y = Y_1 + Y_2 = \frac{1+j\omega R_1C_1}{R_1(2+j\omega R_1C_1)} + \frac{j\omega C_2(1+j\omega R_2C_2)}{1+2j\omega R_2C_2} \qquad 12)$$

We now put $C_1=2C_2$ and $R_1=2R_2$ to allow some simplification.

$$I = I_1 + I_2 = V_{IN}\left(\frac{1}{2R_1(1+j\omega R_1C_2)} - \frac{\omega^2 R_2C_2^{\,2}}{1+j\omega R_1C_2}\right) \qquad 13)$$

$$Y = Y_1 + Y_2 = \frac{1+j\omega R_1C_1}{2R_1(1+j\omega R_1C_2)} + \frac{j\omega C_2(1+j\omega R_2C_2)}{1+j\omega R_1C_2} \qquad 14)$$

We now calculate V_{out} as I/Y:

$$V_{OUT} = \frac{I}{Y} = V_{IN} \frac{\dfrac{\dfrac{1}{2R_1} - \omega^2 R_2 C_2^2}{1 + j\omega R_1 C_2}}{\dfrac{\dfrac{1 + j\omega R_1 C_1}{2R_1} + j\omega C_2 (1 + j\omega R_2 C_2)}{1 + j\omega R_1 C_2}} \qquad 15)$$

By multiplying the numerator and the denominator by $2R_1$ we get:

$$V_{OUT} = V_{IN} \frac{1 - 2\omega^2 R_1 R_2 C_2^2}{1 + j\omega R_1 C_1 + 2j\omega R_1 C_2 (1 + j\omega R_2 C_2)} = V_{IN} \frac{1 - \omega^2 R_1^2 C_2^2}{1 + j\omega 2 R_1 C_1 - \omega^2 R_1 R_2 C_1 C_2} \qquad 16)$$

Finally we get:

$$V_{OUT} = V_{IN} \frac{1 - \omega^2 R_1^2 C_2^2}{1 + 4j\omega R_1 C_2 - \omega^2 R_1^2 C_2^2} \qquad 17)$$

where $\omega_o = 1/(R_1 C_2)$ and $Q = 1/4$